

Multiple Kernel Collaborative Representation Based Classification

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Abstract—At present, collaborative representation based classification (CRC) is widely used in many pattern classification and recognition tasks. Meanwhile, spatial pyramid matching (SPM) method, which considers the spatial information in representing the image, is efficient for image classification. However, for SPM, the weights to evaluate the representation of different subregions are fixed. In this paper, we combine CRC and multiple kernel learning approach, propose the multiple kernel collaborative representation based classification (MKCRC) method, and apply it to image classification to learn the weights of different subregions in representing the image. Experimental results have obvious advantages than state-of-the-art methods in several benchmark datasets, such as Caltech101, 102flowers, Scene 15 and UIUC sports..

Keywords—Image classification, Multiple kernel learning, Spatial pyramid matching

I. INTRODUCTION

Sparse representation is a very attractive research field in many field in past 20 years. In signal processing, we can accurately reconstruct the original signal by using the sparsity of the signal when the sampling rate is lower than traditional Nyquist sampling rate. In statistics, sparse representation applies in lasso model[1], [2] which is the representative model of linear regression. In pattern recognition, sparse representation applies in object detection and classification [3], [4]. Although sparse representation achieves different goals in different areas, the essence of sparse representation is the same: compressing the nature signal and then regarding the signal as a linear combination of finite elements.

Sparse representation based classification(SRC)[5], [6], [7] has been applied in the field of face recognition successfully and has achieved superior performance to several benchmark classifiers. Many researchers focused on the importance of ℓ_1 norm regularized sparse representation based classification. Later research showed that the performance of sparse representation classifier is not decided by sparsity. Zhang *et al.* [8] believed that the classified effect of SRC is actually obtained by using the collaboration of whole training set. They considered the sparse representation based classification is a special case of collaborative representation based classification. That is to say, using linear combination of minimum train set to describe the test sample. The essence of collaborative representation is using the train set of all categories to represent unidentified test

sample together. High performance of collaborative representation based classification (CRC) method has been verified. For example, Zhang *et al.* [9] used CRC method on face data sets to test its classified accuracy and achieved excellent performance.

Recently, the kernel approach[10] has been widely used to many algorithms, such as kernel principal component analysis (KPCA)[11]and kernel fisher discriminant analysis(KFD)[12]. Its core idea is: by using nonlinear mapping to map the original data to an appropriate high dimensional feature space, then analyzing and processing the model by general linear learning machine. Comparing with the traditional model, kernel approach has several obvious advantages: first, general nonlinear learning machine is not easy to react characteristics of specific application problems, but the nonlinear mapping of kernel approach is convenient for the integration of relevant prior knowledge. Furthermore, linear learning machine can be controlled better, so we can guarantee the generalization performance. And the most important point is kernel approach is the way to achieve efficient computation. It uses kernel function to make nonlinear mapping and linear learning machine achieve a synchronous computation, so the the complexity is independent of the dimension of high dimensional feature space.

For image representation, bag of words (BOW) model [13] is an efficient method which describes an image according to statics the frequency of the words (i.e. the visual vocabulary). It can efficiently extract the discriminative information to represent the image. Sparse representation is proposed to add ambiguity during coding to improve the traditional BOW model. Convolutional neural network extremely improved the representation performance to use the deep neural networks. However, all the methods seriously limits the description ability of image representation, due that all these methods loss spatial information. To get over the problem, Lazebnik[14] raised spatial pyramid matching (SPM) model to add spatial information of local feature to BOW model. The proposed method combines the representation of subregions together. The weights to evaluate the representation of different subregions are fixed. The SPM model has achieved excellent performance for image classification.

In this paper, we combine CRC and multiple kernel learning approach, propose the multiple kernel collaborative represen-

tation based classification (MKCRC) method, and apply it to SPM model to learn the weights of different subregions in representing the image. The main contributions is as follows,

1. The paper proposes multiple kernel collaborative representation based classification method. It combines CRC and multiple kernel learning approach. Our proposed MKCRC is capable of evaluating the weights of different kernels.

2. To improve the traditional SPM model, where the weights to evaluate the representation of different subregions are fixed, our proposed MKCRC is applied to SPM model to learn the weights of different subregions.

3. Experimental results have obvious advantages than state-of-the-art methods in some benchmark data sets, such as Caltech101, 102flowers, Scene 15 and UIUC-Sports.

The reminder of this paper is organized as follows: Section II proposes the MKCRC algorithm. Section III presents the optimization scheme. Section IV provides experimental evaluations and comparisons. Section V concludes our work.

II. MULTIPLE KERNEL COLLABORATIVE REPRESENTATION BASED CLASSIFICATION (MKCRC)

A. Spatial pyramid matching model

SPM model has reached good effect in image classification. It is known as a benchmark of image classification method. The essence of SPM is to divide the image on spatial level. Fig.1 shows its principle. Original image is regarded as first layer of pyramid and its label is 0. Then, the original image is divided into four parts. We extract their feature of each subregion, labeling them as 1. Repeating divided process of each subregion, finally, image spatial pyramid matching model is constructed. SPM calculates features in each subregions and connects features of all subregions to a feature vector to describe image.

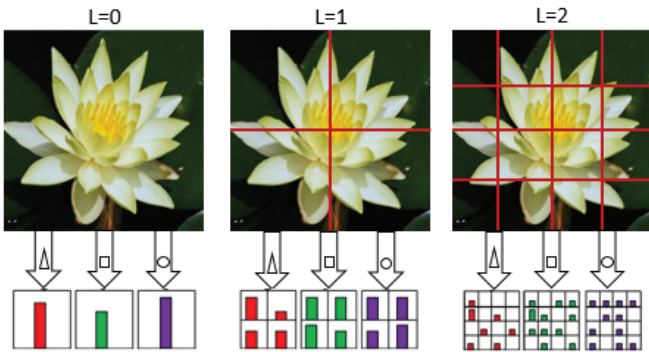


Fig. 1. Schematic diagram of SPM model. Triangle, rectangle and circle in the figure represent different features. With the increase of sub image, the information we extract is more detailed.

B. CRC

Collaborative representation based classification (CRC) can be considered as methods of rearranging the structure of the original data in order to make the representation compact under the given bases (i.e. the training set). Hence, the data vector is represented as a linear combination of bases vectors.

Algorithm 1 Algorithm for CRC

Require: Training samples $X \in \mathbb{R}^{D \times N}$, η , and test sample y

- 1: Code y with the dictionary X via collaborative representation Eqn. (1).
- 2: **for** $c = 1; c \leq C; c++$ **do**
- 3: Compute the residuals $e^c(y) = \|y - X^c \hat{s}^c\|_2^2$
- 4: **end for**
- 5: $id(y) = \arg \min_c \{e^c\}$
- 6: **return** $id(y)$

Zhang *et al.* [8] proposed the collaborative representation based classification (CRC) algorithm for robust image recognition. Specifically, given the training samples $X = [X^1, X^2, \dots, X^C] \in \mathbb{R}^{D \times N}$, $X^c \in \mathbb{R}^{D \times N_c}$ represents the training samples from the c_{th} class, C represents the number of classes, N_c represents the number of training samples in the c_{th} class ($N = \sum_{c=1}^C N_c$), and D represents the dimensions of the samples. Supposing that $y \in \mathbb{R}^{D \times 1}$ is a test sample, the collaborative representation algorithm aims to solve the following objective function:

$$\hat{s} = \arg \min_s \left\{ \|y - Xs\|_2^2 + \eta \|s\|_2^2 \right\}. \quad (1)$$

Here, η is the regularization parameter to control the tradeoff between fitting goodness and collaboration.

The collaborative representation based classifier is to find the minimum value of the residual error for each class:

$$id(y) = \arg \min_c \|y - X^c \hat{s}^c\|_2^2. \quad (2)$$

The procedure of CRC is shown in Algorithm 1. The residual error e_c in Algorithm 1 is associated with most of the images in class c . CRC algorithms directly use the training samples as the dictionary.

C. MKCRC

Suppose that there exists a feature mapping function $\phi : \mathbb{R}^D \rightarrow \mathbb{R}^t$. It maps the original feature space to the high dimensional kernel space: $X = [X^1, X^2, \dots, X^C] \rightarrow \phi(X) = [\phi(X^1), \phi(X^2), \dots, \phi(X^C)]$, $y \rightarrow \phi(y)$. Then, the objective function of Eq. (1) can be generalized to reproducing kernel Hilbert spaces as

$$\hat{s} = \arg \min_s \left\{ \|\phi(y) - \phi(X)s\|_2^2 + \eta \|s\|_2^2 \right\}. \quad (3)$$

where the kernel function $\kappa(x, y) = \phi(x)^T \phi(y)$.

To map the features from original space to a high dimensional space, we usually utilize several kernels and their weight are different (e.g. we consider each subregion as a kernel mapping for SPM model). The weight of each kernel can be learned to achieve superior performance. The mode of multiple kernel by Lanckriet *et al.* [15] is Eq. (4).

$$\sum_{i=1}^m \beta_m \kappa_m(x, y) = \kappa(x, y) \quad (4)$$

We constrain the weight of each kernel as $\sum_{m=1}^M \beta_m^2 = 1$ (M is the number of all kernels). The objective function of our proposed MKCRC is as follows,

$$f(s, \beta) = \left\{ \|\phi(y) - \phi(X)s\|_2^2 + \eta \|s\|_2^2 \right\}$$

$$s.t. \kappa(x, y) = \sum_{m=1}^M \beta_m k_m(x, y), \sum_{m=1}^M \beta_m^2 = 1, \quad (5)$$

where $\kappa(x, y) = \phi(x)^T \phi(y) = \sum_{m=1}^M \{\phi_m(x)^T \phi_m(y)\}$.

III. OPTIMIZATION OF THE OBJECTIVE FUNCTION

A. Optimization of s

When β is fixed, the objective function is,

$$f(s) = \|\phi(y) - \phi(X)s\|_2^2 + \eta \|s\|_2^2 \quad (6)$$

To optimize the Eq. (6), it can be transformed as follows,

$$\begin{aligned} f(s) &= \|\phi(y) - \phi(X)s\|_2^2 + \eta \|s\|_2^2 \\ &= \text{trace}\{\phi(y)^T \phi(y)\} - 2\text{trace}\{\phi(y)^T \phi(X)s\} \\ &\quad + \text{trace}\{s^T \phi(X)^T \phi(X)s\} + \text{trace}\{s^T s\} \\ &= \text{trace}\{\kappa(y, y)\} - 2\text{trace}\{\kappa(y, X)s\} \\ &\quad + \text{trace}\{s^T (\kappa(X, X) + \eta I)s\} \end{aligned} \quad (7)$$

The partial derivative of $f(s)$ to s is,

$$\frac{\partial f(s)}{\partial s} = -2\kappa(y, X)^T + 2[\kappa(X, X) + \eta I]s \quad (8)$$

let $\frac{\partial f(s)}{\partial s} = 0$, we can get the value of s ,

$$s = [\kappa(X, X) + \eta I]^{-1} \kappa(y, X)^T \quad (9)$$

B. Update β

With fixed s , the objective function Eq. (5) is as follows,

$$f(\beta) = \|\phi(y) - \phi(X)s\|_2^2$$

$$s.t. \kappa(x, y) = \sum_{m=1}^M \beta_m k_m(x, y), \sum_{m=1}^M \beta_m^2 = 1, \quad (10)$$

where β is the weights of kernels.

To optimize the objection function Eq. (10), lagrangian multiplier is adopted.

$$g(\lambda, \beta) = f(\beta) + \lambda \left(\sum_{m=1}^M \beta_m^2 - 1 \right) \quad (11)$$

To optimize the Eq. (11), it can be transformed as follows,

$$\begin{aligned} g(\lambda, \beta) &= \text{trace}\left\{ \sum_{m=1}^m \beta_m \kappa_m(y, y) \right\} \\ &\quad - 2\text{trace}\left\{ \sum_{m=1}^m \beta_m \kappa_m(y, X)s \right\} \\ &\quad + \text{trace}\left\{ s^T \sum_{m=1}^m \beta_m \kappa_m(X, X)s \right\} + \lambda \left(\sum_{m=1}^M \beta_m^2 - 1 \right) \end{aligned} \quad (12)$$

Algorithm 2 Algorithm for MKCRC

Require: Training samples $X \in \mathbb{R}^{D \times N}$, η , and test sample y

- 1: Compute $\kappa(X, X)$, $\kappa(y, X)$, get initial β_m and s .
 - 2: Update s by Eq. (9).
 - 3: Update β by Eq. (15).
 - 4: Go back to Update s until the condition of convergence is satisfied.
 - 5: **for** $c = 1; c \leq C; c++$ **do**
 - 6: Compute the residuals $e^c(y) = \|\phi(y) - \phi(X)^c s^c\|_2^2$
 - 7: **end for**
 - 8: $id(y) = \arg \min_c \{e^c\}$
 - 9: **return** $id(y)$
-

The partial derivative of $g(\lambda, \beta)$ to β_m is,

$$\begin{aligned} \frac{\partial g(\lambda, \beta)}{\partial \beta_m} &= \text{trace}\{\kappa_m(y, y)\} - 2\text{trace}\{\kappa_m(y, X)s\} \\ &\quad + \text{trace}\{s^T \kappa_m(X, X)s\} + 2\lambda \beta_m \end{aligned} \quad (13)$$

The partial derivative of $g(\lambda, \beta)$ to λ is,

$$\frac{\partial g(\lambda, \beta)}{\partial \lambda} = \sum_{m=1}^M \beta_m^2 - 1 \quad (14)$$

let $\frac{\partial g(\lambda, \beta)}{\partial \beta_m} = 0$ and $\frac{\partial g(\lambda, \beta)}{\partial \lambda} = 0$, we can get the value of β_m ,

$$\begin{aligned} \beta_m &= 2\text{trace}\{\kappa_m(y, X)s\} - \text{trace}\{\kappa_m(y, y)\} \\ &\quad - \text{trace}\{s^T \kappa_m(X, X)s\} \\ \beta_m &= \frac{\beta_m}{\sqrt{(\beta_1^2 + \beta_2^2 + \dots + \beta_M^2)}} \end{aligned} \quad (15)$$

C. The algorithm of MKCRC

An intuitive interpretation of MKCRC is as Algorithm 2.

IV. EXPERIMENTS AND DISCUSSIONS

In order to verify the effectiveness and robustness of the MKCRC algorithm. In the section, we use several data sets which are the most commonly used in image classification experiments: Calthch101, 102flowers, Scene 15 and UIUC-Sports. To demonstrate the effectiveness of MKCRC, we experiment it on these data sets, respectively. And we further analysis the influence of different feature fusion methods, different layers of pyramid and the weight of each layer. Fig.2 shows our classified target of each image set. We compare our method with ordinary CRC method and sparse representation classification(SRC) method.

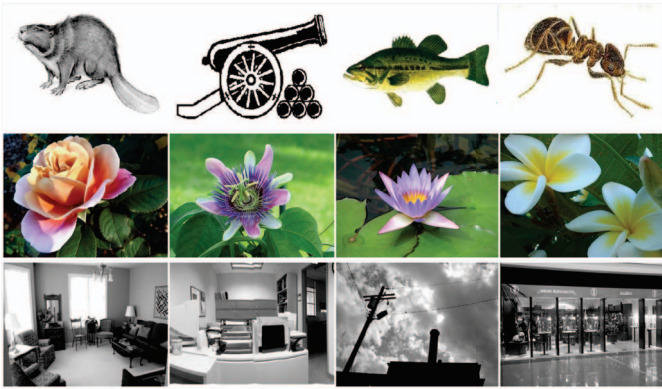


Fig. 2. Three images which belong to different category in the first column, the remaining columns show three train images that produce feature vectors

We carried out 10 trials of each parameter and report average results. The number of train set images is 5 and test set is 10.

A. imagenet-vgg-verydeep-19 model

We use imagenet-vgg-verydeep-19[16] model as our train model. The size of its convolutional filter is 3×3 , which is a very small receptive field and it's an improvement of current technology. The depth of imagenet-vgg-verydeep-19 model is 43, that is to say, it includes 43 working layers. It contains 16 convolutional layers, 18 rectified linear units (ReLU) layers and 5 spatial pooling layers[17]. The function of ReLU nonlinearity is to accelerate training speed and spatial pooling layer is to reduce data dimension after convolution. We use reciprocal fourth layer's vector as the feature with 4096 dimensions. For spatial pyramid matching model, we divide the image into two layers, for the first layer, the subregion is the original image. For the second layer, the image is equally divided into four subregions. Then, the image feature is the combination of these five subregions' representation, i.e. the length of image feature is $4096 \times 5 = 20480$.

B. Experiment results based Caltech101 data set

Caltech101 data set [18] has 101 classes (including animals, transportation, flowers and so on), the number of images of each class range from 31 to 800 and there is a large difference between each category. In Caltech101 data set, target is located in the middle of the image and it is different from background. So it is relatively easy to classify.

Table 1 shows our MKCRC results based Caltech101 data set where η in the table means a parameter to adjust kernel's weight and acc means average accuracy of 10 trials.

Table 1. MKCRC result based Caltech101 data set

η	1/128	1/64	1/32	1/16	1/8	1/4	1/2
acc(%)	84.62	84.68	84.63	84.70	84.78	84.31	83.71

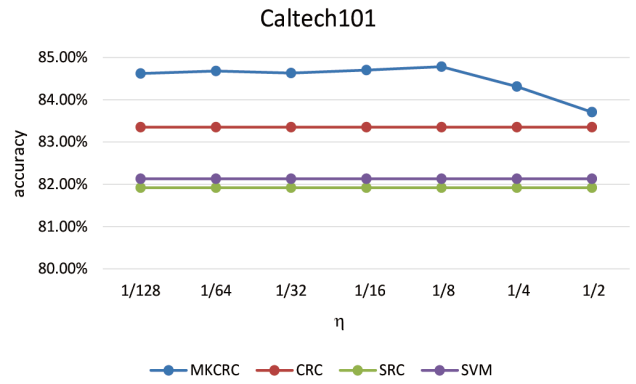


Fig. 3. Average accuracy of SVM, SRC, CRC and MKCRC method. In the figure, we choose the best performance of CRC, SRC and SVM. The best performance of CRC is 83.35%, SRC is 81.92% and SVM is 82.13%.

According to Fig.3, we know that when $\eta=1/8$, MKCRC owns the highest classified accuracy 84.78%. It is higher 1.43% than CRC method.

C. Experiment results based 102flowers data set

102flowers data set [19] includes 102 different type of flowers, most of them are from British. And the number of images of each class ranges from 40 to 258. Table 2 shows MKCRC results based 102flowers data set.

Table 2. MKCRC result based 102flowers data set

η	1/128	1/64	1/32	1/16	1/8	1/4	1/2
acc(%)	73.54	73.75	73.80	74.21	74.27	73.75	72.75

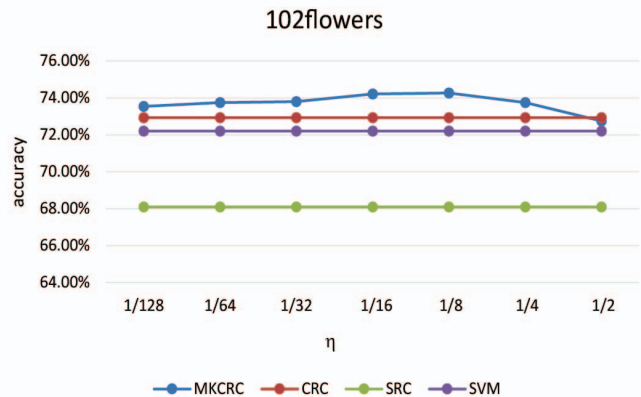


Fig. 4. Average recognition rate of SVM, SRC, CRC and MKSRC. Like the result of Caltech101 data set, we select the best results of SVM, SRC and CRC methods. In the figure, the best performance of CRC is 72.93%, SRC is 68.09% and SVM is 72.20%.

We can know that the highest classified accuracy of MKCRC is 74.27%. At the same time, $\eta=1/16$. It is higher 1.34% than CRC method.

D. Experiment results based Scene 15 data set

Scene 15 data set [14] contains 15 categories, the number of images of each class ranges from 200 to 400. Table 3 shows MKCRC results based Scene 15 data set.

Table 3. MKCRC result based Scene 15 data set

η	1/128	1/64	1/32	1/16	1/8	1/4	1/2
acc(%)	78.60	78.47	78.40	78.47	78.80	78.80	78.60

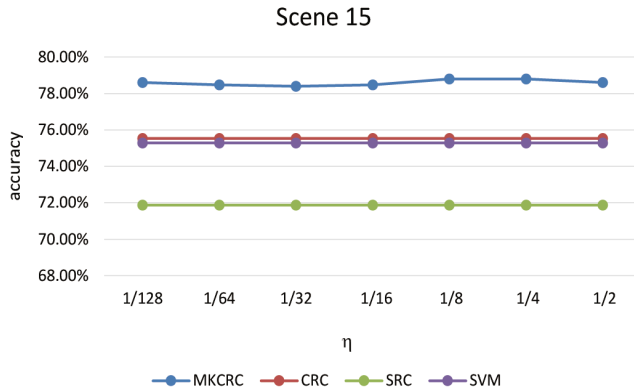


Fig. 5. Average recognition rates of SVM, SRC, CRC and MKSRC. In the figure, the average accuracy of CRC is 75.53%, SRC is 71.87% and SVM is 75.28%.

When $\eta=1/8$ and $1/4$, MKCRC method owns the best results and the highest classified accuracy is 78.80%, it is higher 3.27% than CRC method.

E. Experiment results based UIUC-Sports data set

UIUC-Sports data set [20] is widely used in image classification which contains 8 sports event categories (including running, snow boarding and so on). According to human subject judgement, images are divided into easy and medium. And average size of each image is about 200 pixels. Table 4 shows MKCRC results based UIUC-Sports data set.

Table 4. MKCRC result based UIUC-Sports data set

η	1/128	1/64	1/32	1/16	1/8	1/4	1/2
acc(%)	88.00	87.88	87.88	88.00	88.25	88.25	87.50

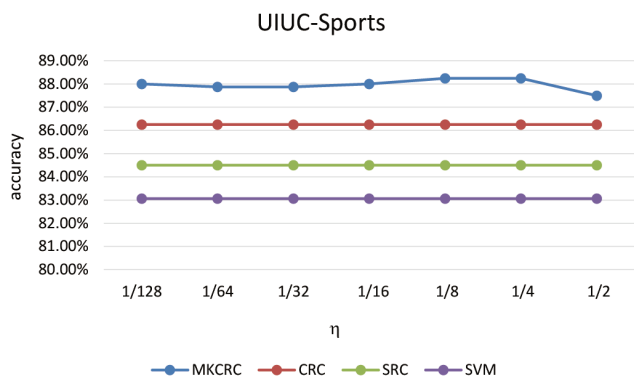


Fig. 6. Average recognition accuracy of SVM, SRC, CRC and MKSRC. In the figure, the average accuracy of CRC is 86.25%, SRC is 84.50% and SVM is 83.06%.

MKCRC's best results is higher 2% than CRC method, higher 3.75% than SRC method and 5.19% than SVM method when $\eta=1/128$.

V. CONCLUSION

The paper proposed a method named multiple kernel collaborative representation based classification and its validity has been demonstrated by some experiments. Core idea of MKCRC method is to learn the weights or contributions of each subregion in SPM model. Thus, the MKCRC method is applied to the spatial pyramid matching model to improve the image classification performance. We experiment it on some data sets and its recognition accuracy is better than common methods, such as CRC, SRC and SVM. In the future, we will devote to improving MKCRC method to achieve higher accuracy by estimating the kernel parameter.

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